Estimating Market Risk Measures

估计市场风险计量

Estimating Returns

Profit/loss data: Change in value of asset/portfolio, P_t, at the end of period t plus any interim payments, D_t.

$$P/L_t = P_t + D_t - P_{t-1}$$

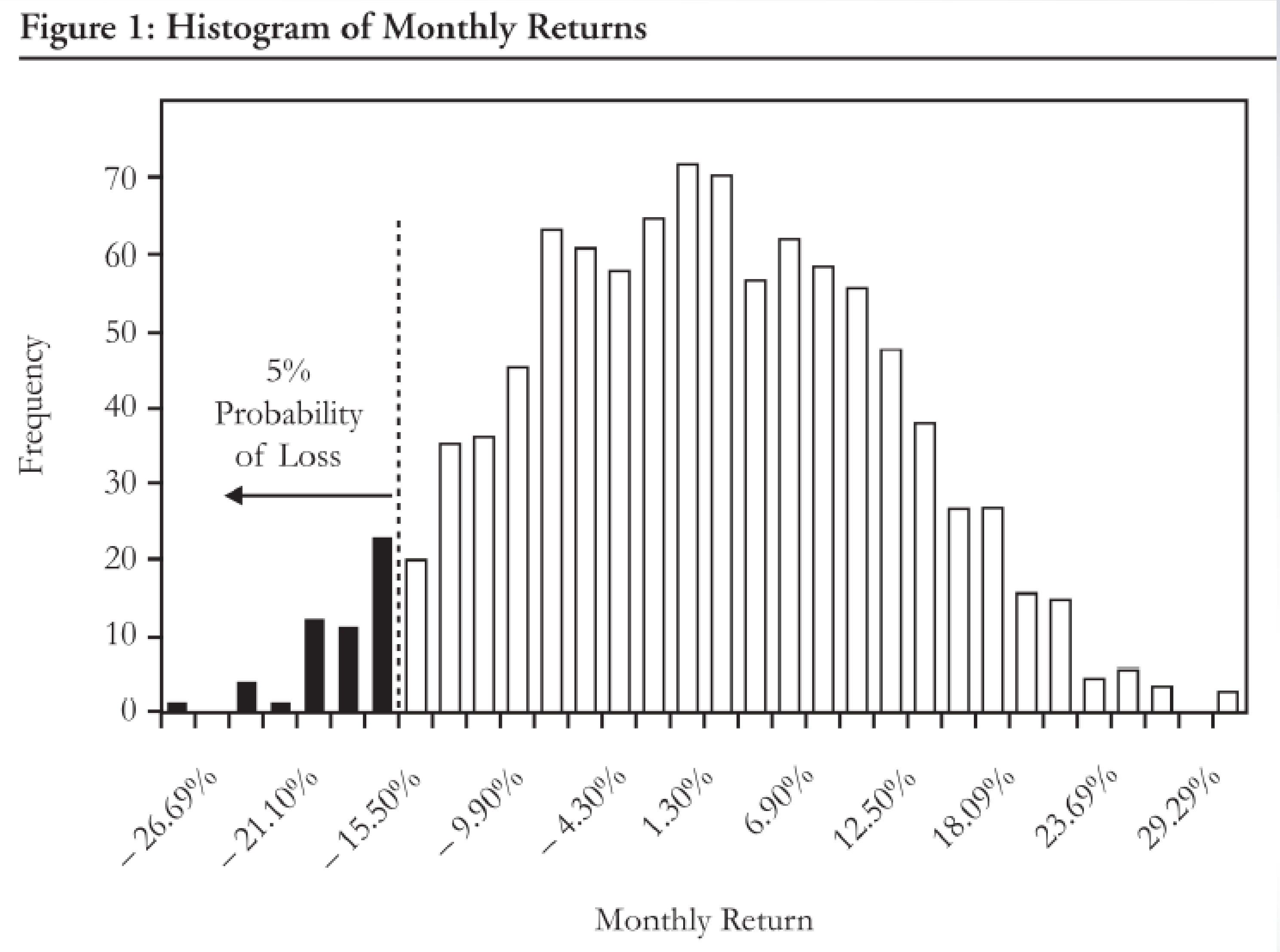
Arithmetic return data: Assumption is that interim payments do not earn a return (i.e., no reinvestment). Hence, this approach is not appropriate for long investment horizons.

$$r_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} = \frac{P_t + D_t}{P_{t-1}} - 1$$

Geometric return data: Assumption is that interim payments are continuously reinvested. Note that this approach ensures that asset price can never be negative.

$$R_t = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right)$$

Calculate VaR using a Historical Simulation Approach



Calculate VaR using a Historical Simulation Approach

Example: Identifying the VaR limit

Identify the ordered observation in a sample of 1,000 data points that corresponds to VaR at a 95% confidence level.

Answer:

Since VaR is to be estimated at 95% confidence, this means that 5% (i.e., 50) of the ordered observations would fall in the tail of the distribution. Therefore, the 51st ordered loss observation would separate the 5% of largest losses from the remaining 95% of returns.

Calculate VaR using a Historical Simulation Approach

Professor's Note: VaR is the quantile that separates the tail from the body of the distribution. With 1,000 observations at a 95% confidence level, there is a certain level of arbitrariness in how the ordered observations relate to VaR. In other words, should VaR be the 50th observation (i.e., $\alpha \times n$), the 51st observation [i.e., $(\alpha \times n) + 1$], or some combination of these observations? In this example, using the 51st observation was the approximation for VaR, and the method used in the assigned reading. However, on past FRM exams, VaR using the historical simulation method has been calculated as just: $(\alpha \times n)$, in this case, as the 50th observation.

Calculate VaR using a Historical Simulation Approach

Example: Computing VaR

A long history of profit/loss data closely approximates a standard normal distribution (mean equals zero; standard deviation equals one). **Estimate** the 5% VaR using the historical simulation approach.

Answer:

The VaR limit will be at the observation that separates the tail loss with area equal to 5% from the remainder of the distribution. Since the distribution is closely approximated by the standard normal distribution, the VaR is 1.65 (5% critical value from the *z*-table). Recall that since VaR is a one-tailed test, the entire significance level of 5% is in the left tail of the returns distribution.

Parametric Estimation Approach

- **Normal VaR**

$$VaR(\alpha\%) = -\mu_{P/L} + \sigma_{P/L} \times z_{\alpha}$$

Example: Computing VaR (normal distribution)

Assume that the profit/loss distribution for XYZ is normally distributed with an annual mean of \$15 million and a standard deviation of \$10 million. **Calculate** the VaR at the 95% and 99% confidence levels using a parametric approach.

Answer:

VaR(5%) = –\$15 million + \$10 million × 1.65 = \$1.5 million. Therefore, XYZ expects to lose at most \$1.5 million over the next year with 95% confidence. Equivalently, XYZ expects to lose more than \$1.5 million with a 5% probability.

VaR(1%) = –\$15 million + \$10 million × 2.33 = \$8.3 million. Note that the VaR (at 99% confidence) is greater than the VaR (at 95% confidence) as follows from the definition of value at risk.

Parametric Estimation Approach

- Now suppose that the data you are using is **arithmetic return data** rather than profit/loss data.

$$VaR(\alpha\%) = \left(-\mu_r + \sigma_r \times z_\alpha\right) \times P_{t-1}$$

Example: Computing VaR (arithmetic returns)

A portfolio has a beginning period value of \$100. The arithmetic returns follow a normal distribution with a mean of 10% and a standard deviation of 20%. **Calculate** VaR at both the 95% and 99% confidence levels.

Answer:

$$VaR(5\%) = (-10\% + 1.65 \times 20\%) \times 100 = \$23.0$$

$$VaR(1\%) = (-10\% + 2.33 \times 20\%) \times 100 = \$36.6$$

Parametric Estimation Approach

- **Lognormal VaR**

$$\text{VaR}(\alpha\%) = P_{t-1} \times \left(1 - e^{\mu_R - \sigma_R \times z_\alpha}\right)$$

Example: Computing VaR (lognormal distribution)

A diversified portfolio exhibits a normally distributed geometric return with mean and standard deviation of 10% and 20%, respectively. **Calculate** the 5% and 1% lognormal VaR assuming the beginning period portfolio value is $100.

Answer:

$$\begin{aligned}\text{Lognormal VaR}(5\%) &= 100 \times (1 - \exp[0.1 - 0.2 \times 1.65]) \\ &= 100 \times (1 - \exp[-0.23]) \\ &= \$20.55\end{aligned}$$

$$\begin{aligned}\text{Lognormal VaR}(1\%) &= 100 \times (1 - \exp[0.1 - 0.2 \times 2.33]) \\ &= 100 \times (1 - \exp[-0.366]) \\ &= \$30.65\end{aligned}$$

Expected Shortfall

- The **expected shortfall** (ES) provides an estimate of the tail loss by **averaging** the VaRs for increasing confidence levels in the tail

Figure 2: Estimating Expected Shortfall

Confidence level	*VaR*	*Difference*
96%	1.7507	
97%	1.8808	0.1301
98%	2.0537	0.1729
99%	2.3263	0.2726
Average	2.003	
Theoretical true value	2.063	

VaR and ES

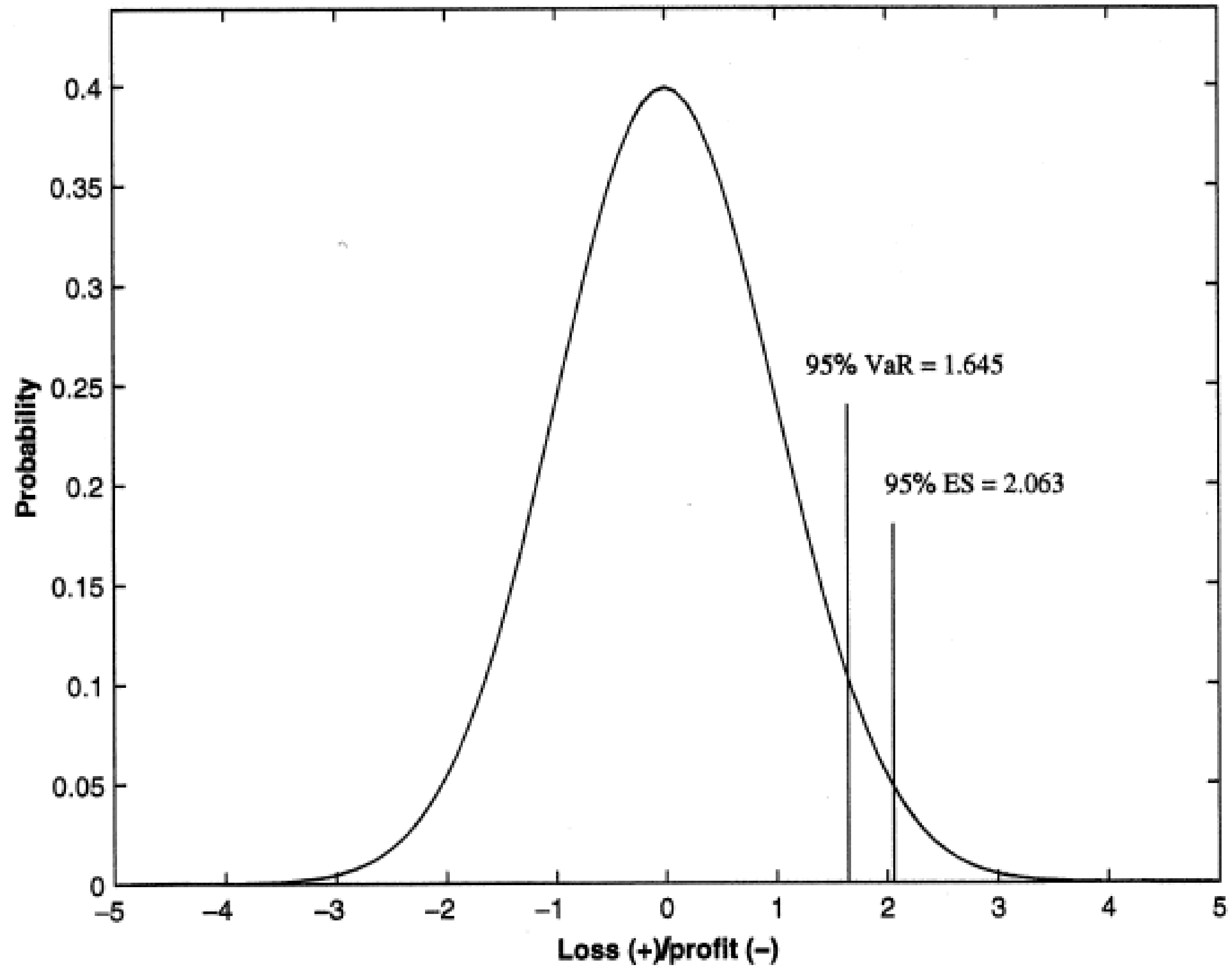


FIGURE 1-8 Normal VaR and ES.

Note: Estimated with the mean and standard deviation of P/L equal to 0 and 1 respectively, using the 'normalesfigure' function.

Estimating Coherent Risk Measures

- A more general risk measure than either VaR or ES is known as a **coherent risk measure.**
- A coherent risk measure is a weighted average of the quantiles of the loss distribution where the weights are **user-specific based on individual risk aversion**. A coherent risk measure will assign each quantile (not just tail quantiles) a weight. The average of the weighted VaRs is the estimated loss.

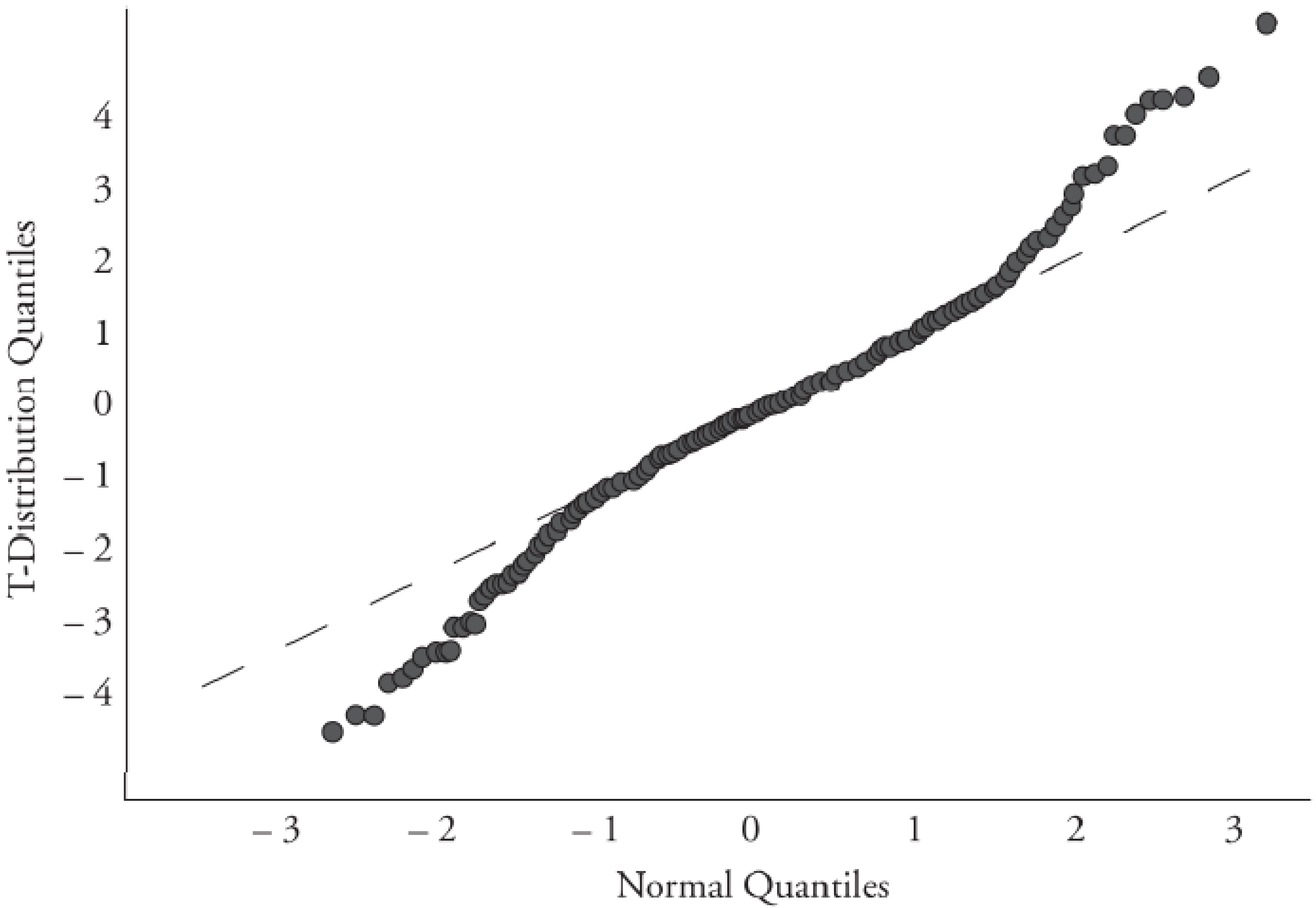
TABLE 1-3 Estimating Exponential Spectral Risk Measure as a Weighted Average of VaRs

Confidence level (α)	αVaR	Weight $\phi(\alpha)$	$\phi(\alpha) \times \alpha VaR$
10%	−1.2816	0	0.0000
20%	−0.8416	0	0.0000
30%	−0.5244	0	0.0000
40%	−0.2533	0.0001	0.0000
50%	0	0.0009	0.0000
60%	0.2533	0.0067	0.0017
70%	0.5244	0.0496	0.0260
80%	0.8416	0.3663	0.3083
90%	1.2816	2.7067	3.4689
Risk measure = mean ($\phi(\alpha)$ times αVaR) =			0.4226

Note: VaRs estimated assuming the mean and standard deviation of losses are 0 and 1,

Quantile - Quantile Plots

Figure 3: QQ Plot



Examples

1. The VaR at a 95% confidence level is estimated to be 1.56 from a historical simulation of 1,000 observations. Which of the following statements is most likely true?
   A. The parametric assumption of normal returns is correct.
   B. The parametric assumption of lognormal returns is correct.
   C. The historical distribution has fatter tails than a normal distribution.
   D. The historical distribution has thinner tails than a normal distribution.

1. D The historical simulation indicates that the 5% tail loss begins at 1.56, which is less than the 1.65 predicted by a standard normal distribution. Therefore, the historical simulation has thinner tails than a standard normal distribution.

Examples

2. Assume the profit/loss distribution for XYZ is normally distributed with an annual mean of $20 million and a standard deviation of $10 million. The 5% VaR is calculated and interpreted as which of the following statements?
   A. 5% probability of losses of at least $3.50 million.
   B. 5% probability of earnings of at least $3.50 million.
   C. 95% probability of losses of at least $3.50 million.
   D. 95% probability of earnings of at least $3.50 million.

2. D The value at risk calculation at 95% confidence is: -20 million + 1.65×10 million = $-\$3.50$ million. Therefore, XYZ is expected to lose at least $-\$3.50$ million over the next year. Since the expected loss is negative, the interpretation is that XYZ will earn less than $3.50 million with 5% probability, which is equivalent to XYZ earning at least $3.50 million with 95% probability.

Examples

3. Which of the following statements about expected shortfall estimates and coherent risk measures are true?

   A. Expected shortfall and coherent risk measures estimate quantiles for the entire loss distribution.

   B. Expected shortfall and coherent risk measures estimate quantiles for the tail region.

   C. Expected shortfall estimates quantiles for the tail region and coherent risk measures estimate quantiles for the non-tail region only.

   D. Expected shortfall estimates quantiles for the entire distribution and coherent risk measures estimate quantiles for the tail region only.

3. **B** ES estimates quantiles for n – 1 equal probability masses in the tail region only. The coherent risk measure estimates quantiles for the entire distribution including the tail region.

Examples

5. The quantile-quantile plot is best used for what purpose?
   A. Testing an empirical distribution from a theoretical distribution.
   B. Testing a theoretical distribution from an empirical distribution.
   C. Identifying an empirical distribution from a theoretical distribution.
   D. Identifying a theoretical distribution from an empirical distribution.

5. C Once a sample is obtained, it can be compared to a reference distribution for possible identification. The QQ plot maps the quantiles one to one. If the relationship is close to linear, then a match for the empirical distribution is found. The QQ plot is used for visual inspection only without any formal statistical test.

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